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PROBLEM SECTION

Through Volume 90 (3-4) 2021 of The Mathematics Student, we had invited solutions from readers to the Problems 1, 2, 3 and 5 mentioned in MS 89 (3-4) 2020, solutions to the problems 1, 2, 4 and 5 mentioned in MS 90 (1-2) 2021, as well as solutions to the ten new problems till January 10, 2022.

As regards to solutions to the four Problems mentioned in MS 89(3-4) 2020, we did not receive any solution to any of the four problems. Therefore solutions provided by Prof. B. Sury, the proposer of the problems, are being published in this section.

As far as solutions to the four Problems mentioned in MS 90 (1-2) 2021 are concerned, we have not received received any solution to any of the problems from the readers of the Mathematics Student. However, we feel that several readers can provide solutions to these problems. Hence we give one more opportunity to the readers to provide their solutions to these problems until April 20, 2022.

As far as solutions to the ten new problems mentioned in MS 90 (3-4) 2021 are concerned, we received one correct solution to Problem 1 which will be printed in this section. Two correct solutions have been received to Problem 7 and we publish the solution which is more precise and elegant.

We pose **Twelve new problems** in this section. We invite Solutions from the readers to the Problems 1, 2 4 and 5 of MS 90 (1-2) 2021, solutions to the remaining eight problems of MS 90 (3-4) 2021 and solutions to the Twelve new problems till April 20, 2022. Correct solutions received from the readers by this date will be published in Volume 91 (3-4) 2022 of The Mathematics Student. This volume is scheduled to be published in May 2022.

New Problems.

Ilir Demiri and **Prof. Shpetim Rexhepi**, Mother Teresa University, Skopje, North Macedonia proposed the following three problems.

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MS 91 (1-2) 2022: Problem 1. Prove that

$$\int_0^1 \left(t^{1/n} - t^{1-1/n}\right)^{n-1} dt = \frac{n}{(1+1/n)\,(1+1/2n)\ldots(1+1/n(n+1))}$$

,
for $n \in \mathbb{N}$.

MS 91 (1-2) 2022 : Problem 2. For the beta function, prove with usual meaning that

$$\sum_{n=0}^{\infty} (B(2,n) - B(3,n)) = \frac{1}{2} \text{ where } n \in \mathbb{N}.$$

MS 91 (1-2) 2022 : Problem 3. For the beta function, prove with usual meaning that

$$B(k,n) = \frac{(k-1)B(k-1,n)}{n+k-1} \text{ where } k, n \in \mathbb{N}.$$

The following five problems have been proposed by **Prof. B. Sury**, Indian Statistical Institute, Bangalore.

MS 91 (1-2) 2022 : Problem 4. Let $f(x) = cot(x)$. Note that $f'(x) = -f(x)^2 - 1$. More generally, for each $n \geq 0$, write

$$n! f(x)^{n+1} = a_n + b_{n0} f(x) + b_{n1} f'(x) + \cdots b_{nn} f^{(n)}(x)$$

for some a_n's and b_{nm}'s. Find a recursion for the b_{ij}'s and determine all the a_n's.

MS 91 (1-2) 2022 : Problem 5. Let a_n denote the number of different ways of putting brackets on a sequence of n objects. For example,

$$(p_1p_2p_3), (p_1)(p_2p_3), (p_1p_2)(p_3)$$

shows that $a_3 = 3$. Check for instance that $a_4 = 11$. Consider also the number b_n of paths from $(0,0)$ to (n,n) which never go above the line $y = x$ and where each step is a unit north, east or north-east (that is, cross). For instance, if we write h, v, c respectively for an eastward horizontal step, a northward vertical step and a southwest-northeast cross step, then

$$hvc, hcv, hhvv, hvhv, cc, chv$$