The Mathematics Student ISSN: 0025-5742
Vol. 91, Nos. 3-4, July-December (2022), 217–229

MS 91 (3-4) 2022: Problem 2. Let $\{a_1, a_2, \cdots, a_n\}$ and $\{b_1, b_2, \cdots, b_n\}$ be two permutations of $\{1, 2, \cdots, n\}$. Show that the set $\{a_1b_1, a_2b_2, \cdots, a_nb_n\}$ does not form a complete residue system modulo n.

Dr. Mohsen Soltanifar, University of Toronto, Canada proposed the following two problems.

MS 91 (3-4) 2022: Problem 3. Let X be a real valued random variable on the real line with finite mean. Assume for some $-\infty < \alpha < \infty$ we have:

$$E(\min(X, \alpha)) = E(\max(X, \alpha)).$$

Calculate the distribution of X.

MS 91 (3-4) 2022: Problem 4. Let $X_1, \cdots, X_n$ be i.i.d random variables with common uniform distribution on $(0, 1)$. Let $p, q > 0$ and define a random variable:

$$S_n(p, q) = (\prod_{i=1}^{n} X_i^p)^{\frac{1}{n^q}}, \quad (n \geq 1).$$

Compute

$$\lim_{n\to\infty} S_n(p, q)$$

if it exists and find values of $p,\ q$ for which it does.

Yathiraj Sharma, M. V. Sarada Vilas College, Mysuru, Karnataka suggested the next problem.

MS 91 (3-4) 2022: Problem 5. Consider the sequence $d_n = 3n + 1$. Prove that the sum of the Legendre symbols $(k/7)$ as k runs through divisors of 12 (7d-4) is 0 whenever $d \neq d_n$. Show further that for infinitely many (but not all) n, the sum is not 0 as k runs over divisors of $12(7d_n - 4)$.

Prof. Shpetim Rexhepi and Ilir Demiri, Mother Teresa University, Skopje, North Macedonia proposed the following two problems.

MS 91 (3-4) 2022: Problem 6. Prove that

$$\int_0^{\infty} \frac{u^3 du}{e^{\sqrt[4]{\frac{15}{4}}u} - 1} = \frac{4\pi^4}{225}.$$

MS 91 (3-4) 2022: Problem 7. For $a> b>e$, e-Euler number, prove that

$$\frac{ln\Gamma(b^a)}{ln\Gamma(a^b)} > \frac{lnb}{lna}.$$

Mr. Toyesh Prakash Sharma of Agra College, Agra suggested the folowing problem.

MS 91 (3-4) 2022: Problem 8. If $n > 0$ and α is the positive root of quadratic equation $x^2 - x - 1 = 0$ then show that the following inequality

$$F_n\alpha^{F_n} + L_n\alpha^{L_n} \geq 2F_{n+1}\alpha^{F_{n+1}}$$

holds.

Further, obtain the above inequality using the convexity of a suitable function where the Fibonacci numbers F_n and the Lucas numbers L_n satisfy the conditiond.

$$F_{n+2} = F_{n+1} + F_n \ , \quad F_0 = 0, \ F_1 = 1;$$

$$L_{n+2} = L_{n+1} + L_n \ , \quad L_0 = 2, \ L_1 = 1.$$

There was a mistake in Problem 1 of MS 91 (1-2) 2022. The correct problem is produced here.

MS 91 (3-4) 2022: Problem 1. (Proposed by Demiri and Rexhepi)

Prove that

$$\int_0^1 \left(t^{\frac{-1}{n}} - t^{1-\frac{1}{n}}\right)^{n-1} dt = \frac{n^n}{(n+1)(n+\frac{1}{2})(n+\frac{1}{3})...(n+\frac{1}{n-1})}$$

where $n \in \mathbb{N}$ *and* $n > 1$.

Solutions to the Old Problems

MS 90 (1-2) 2021: Problem 2. (Proposed by Prof. B. Sury, ISI, Bangalore)

Let $f\colon [0,1] \to \mathbb{R}$ be differentiable, and let $f(0) = 0, f(1) = 1$. Prove that there exist $t_1, \ldots, t_{2021} \in [0,1]$ such that $2021 = \sum_{i=1}^{2021} \frac{1}{f'(t_i)}$.

Dr. Henry Ricardo, Westchester Area Math Circle, New York, USA provided a solution to the problem as given below.